Auxiliary Vector Selection Algorithms For Adaptive Beamforming

Benjamin L Mathews* [1,2], Lamine Mili[1], and Amir I. Zaghloul[1]

[1] Virginia Polytechnic Institute and State University

[2] Northrop Grumman - TASC, Chantilly, VA

The Auxiliary Vector (AV) algorithm iteratively generates a sequence of filters that converge to the Minimum Variance Distortionless Response filter. The early, nonasymptotic elements generated by this algorithm offer favorable bias/variance characteristics and outperform in mean-square filter estimation error, filters generated by other iterative methods. This paper develops two new algorithms for selecting the best AV filter: a MMSE method that can either utilize a training sequence or can operate in a blind decision-directed mode, and a cyclostationary method that exploits the property that cyclostationary signals generate spectral lines when certain nonlinear transformations are applied to them. These new methods are simulated along with previously derived methods in an adaptive beamforming application, and compared with other common beamforming algorithms.

1 Introduction

As the frequency spectrum becomes more crowded, employment of antenna array beamforming techniques has become increasingly necessary to overcome the deleterious effects of co-channel interference, multipath interference, and jamming. One such technique for array beamforming is the Minimum Variance Distortionless Response (MVDR) filter, which seeks to minimize the variance at its output, while simultaneously maintaining a distortionless response towards a desired direction of interest. For a random, zero-mean complex-valued input vector $\mathbf{r}$ of dimension N×1, where N is the number of antenna array elements, and a N×1 steering vector $\mathbf{s}$ which points towards a direction of interest, the MVDR filter can be calculated as [1],

$$\mathbf{w}_{MVDR} = \frac{\mathbf{R}^{-1}\mathbf{s}}{\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}} \tag{1}$$

where $\mathbf{R} = E[\mathbf{r}\mathbf{r}^H]$ is the autocorrelation matrix of the signals impinging upon the array elements and $()^H$ denotes the Hermitian transpose. In most problems the autocorrelation matrix $\mathbf{R}$ can be assumed unknown and must be estimated from available data. A common estimator of $\mathbf{R}$ is the Sample Matrix Inversion (SMI) estimate,

$$\hat{\mathbf{R}}(M) = \frac{1}{M}\sum_{m=1}^{M}\mathbf{r}\mathbf{r}^H \tag{2}$$

where M is the number of snapshots used in estimating $\mathbf{R}$. Substituting $\hat{\mathbf{R}}(M)$ for $\mathbf{R}$, we form a new estimator for the MVDR filter,

$$\hat{\mathbf{w}}_{MVDR}(M) = \frac{\hat{\mathbf{R}}(M)^{-1}\mathbf{s}}{\mathbf{s}^H\hat{\mathbf{R}}(M)^{-1}\mathbf{s}} \tag{3}$$

From this point on, unless otherwise mentioned, we will assume that all of our covariance matrices have been generated using the SMI method.

An iterative Auxiliary Vector (AV) algorithm for calculation of the MVDR filter has been proposed in [2] that requires no explicit matrix inversions or eigendecompositions. This algorithm starts with a steering vector towards a direction of interest, and produces a series of filters that converge to the MVDR filter. It has been shown that the starting with the initial iteration of the algorithm, the filter estimator bias rapidly decays to zero, while the estimator covariance asymptotically converges at a much slower rate. The implication of this is that after relatively few iterations, the filter estimator bias has fallen to nearly zero, while the filter covariance is still relatively small. At some relatively small number of iterations n (which is the parameter we seek to estimate in this paper), the output of the AV algorithm is optimum in the MS estimation error sense ($E[||\hat{\mathbf{w}}_n(M) - \mathbf{w}_{MVDR}||^2]$), where $\mathbf{w}_{MVDR}$ is the ideal weight vector from equation 1, calculated from the ideal, known covariance matrix.

2 Background

The AV algorithm starts with a steering vector towards a direction of interest, and produces a series of filters [2],

$$\hat{\mathbf{w}}_N = \left(\frac{\rho^*}{||\mathbf{s}||^2}\right)\mathbf{s} - \sum_{n=1}^{N-1}\mu_n\mathbf{g}_n \tag{4}$$

that converge to the MVDR filter as $N \to \infty$. The scalar μ_n,

$$\mu_n = \frac{\mathbf{g}_n^H\mathbf{R}\mathbf{w}_{n-1}}{\mathbf{g}_n^H\mathbf{R}\mathbf{g}_n} \tag{5}$$

minimizes the MS error between the output of the the previous iteration of the filter, $\mathbf{w}_{n-1}\mathbf{r}$, and $\mu_n^*\mathbf{g}_n^H\mathbf{r}$, where $\mathbf{g}_n$ is referred to as the auxiliary vector. The vector $\mathbf{g}_n$,

$$\mathbf{g}_n = \left(\mathbf{I} - \frac{\mathbf{s}\mathbf{s}^H}{||\mathbf{s}||^2}\right)\mathbf{R}\mathbf{w}_{n-1} \tag{6}$$

is orthonormal with respect to $\mathbf{s}$, is chosen to maximize the cross-correlation magnitude $|E[(\mathbf{w}_{n-1}^H\mathbf{r})(\mathbf{g}_n^H\mathbf{r})^*]|$.

The problem we seek to solve is to find the number of iterations, $n = N_{opt}$, at which $E[||\hat{\mathbf{w}}_n(M) - \mathbf{w}_{MVDR}||^2]$ is minimized. The problem of selecting the appropriate number of iterations has been explored in previous works [3] for several specific cases and applications. In this paper we explore two more selection algorithms. The first algorithm exploits the cyclostationity present in digitally modulated signals and utilizes a cost function that is based on second-order statistics. The second is a MMSE algorithm that can operate either using a training sequence, or blindly using a decision directed algorithm.

3 The Proposed Cyclostationarity-Based Filter Selection Algorithm

This selection algorithm exploits the fact that cyclostationary signals generate spectral lines when they pass through the nonlinear transformation $()^p$, and is similar to the algorithm used to directly compute the MVDR weight vector in [4]. If the signal of interest is digitally modulated and is of a known type, p may be chosen appropriately, and a filter selection rule can be used that chooses the AV iteration that minimizes the mean square error between the array output using that filter iteration, and a reference complex exponential. Mathematically, we define the rule as,

$$n_{\text{cyclo}} = \arg\min_n \left\{ \sum_{m=1}^{M} |e^{j2\pi\alpha t} - x_{m,n}^p(t)|^2 \right\} \tag{7}$$

where the order of the nonlinearity p and the frequency α at which we expect to see a spectral line are selected based on the signal type, sample rate, and baud rate. In the case of Binary Phase Shift Keying, we expect to see a spectral lines at $f_c \pm m f_d$, where f_c is the carrier frequency, f_d is the baud rate, and m is a real-valued integer [5]. The output of this cost function is the MSE between the complex exponential at frequency α and the output of the array using the n-th AV filter, after the $()^p$ transformation. As we can see, this algorithm requires no training sequence, and if p is chosen to be sufficiently high, this algorithm can operate without a priori knowledge of the type of modulation used in the signal of interest.

4 The Proposed MMSE Filter Selection Algorithm

If the signal of interest is digitally modulated and its type is known a priori, a cost function can be created to determine the optimum number of iterations in the MMSE sense. The algorithm chooses the filter iteration that yields the minimum cumulative Euclidian distance between the received constellation points, and the known or estimated points. Assuming baud-synchronous resampling on the antenna array, for a sequence of M constellation points x_0, x_1, ... x_{M-1}, the MMSE decision rule is,

$$n_{\text{MMSE}} = \arg\min_n \left\{ \sum_{m=1}^{M} \hat{x}_{m,n} - x_m \right\} \tag{8}$$

where $x_{m,n} = \mathbf{w}_n^H \mathbf{r}_m$ is the m-th received snapshot filtered with the output of the n-th stage of the AV filter, and x_m can either be a known constellation point, in the case of a known training sequence, or it can be estimated as the point in the known constellation with the minimum Euclidian distance from the received point $\hat{x}_m$.

5 Simulation Results

To demonstrate the performance of the proposed AV algorithm selection rules, we simulate the reception of a binary phase shift keying (BPSK) modulated signal in the presence of three BPSK interferers and AWGN with a N=10 element linear antenna array, with an element spacing of $\lambda/2$. We form a covariance matrix with 64 snapshots from the antenna array and generate the AV filter according the two rules that we have developed in this paper, and according to the Cross-Validated Minimum Output Variance rule and the J-Divergence rule, both proposed in [3].

Fig. 1 shows the array responses of the SMI MVDR beamformer from equation 3, and the ideal AV beamformer, which we arrive at on the 17th iteration. We

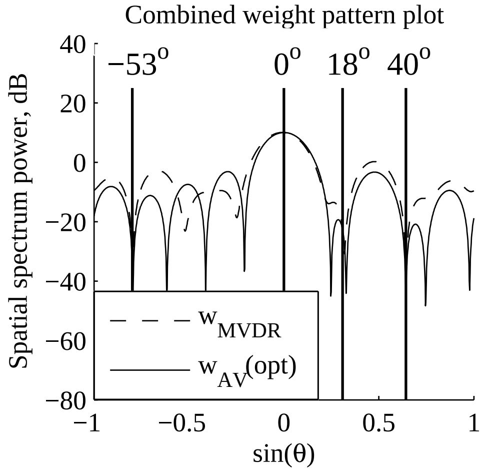

Figure 1: Array Responses

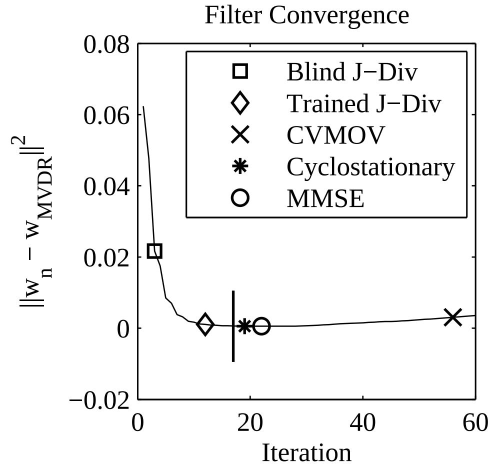

Figure 2: Filter Selection Performance

note that as we continue iterating through the AV algorithm, the array response of the AV beamformer will converge to that of the SMI MVDR beamformer. In Fig. 2 we can see that both proposed methods are comparable to the trained J-Divergence method, and both clearly outperform the blind J-divergence and the CVMOV methods.

6 Conclusions

This paper has presented two new methods for selection of the optimum number of AV filter iterations. Simulations demonstrated the effectiveness of the AV algorithm in an adaptive beamforming application, and the ability of both selection methods to accurately choose the optimum number of AV algorithm iterations.

References

[1] Haykin, S., *Adaptive Filter Theory*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1991.

[2] Pados, D. A. and Karystinos, G. N., An iterative algorithm for the computation of the MVDR filter. *IEEE Trans. Signal Process.* **49** (2001), 290-300.

[3] Qian, H. and Batalama, S. N., Data-record-based criteria for the selection of an auxiliary-vector estimator of the MVDR filter. In *Proc. Asilomar Conf. Signals, Systems.*, Computers. pp. 802- 807, Pacific Grove, CA, Oct. 2000.

[4] Castedo, L. and Figueiras-Vidal, A.R., An adaptive beamforming technique based on cyclostationary signal properties. *IEEE Trans. Signal Process.* **43** (1995), 1637-1650

[5] Gardner, W.A., Cyclostationarity in Communications and Signal Processing (New York: IEEE Press, 1994).